Learning to Solve PDE-constrained Inverse Problems with Graph Networks

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Abstract & Background

Inferring knowledge from observation data by leveraging simulation and mathematical models (PDEs) are important for many application domains across science and engineering

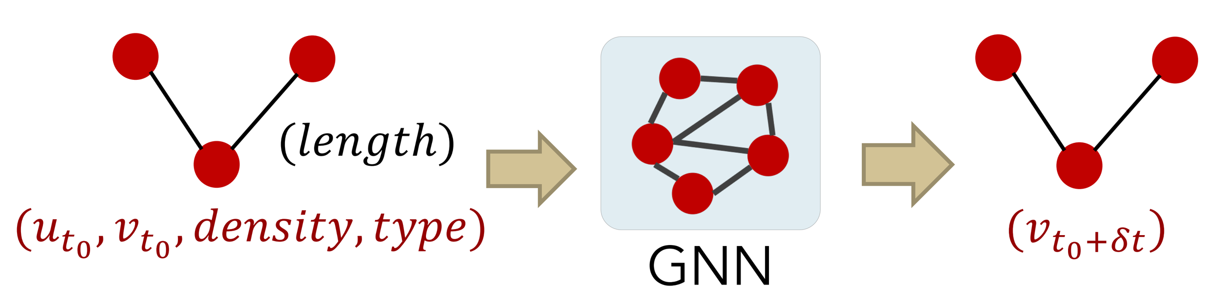
Sparse Observations Underlying PDEs (partial differential equations)	⇨	Internal structures, Physics parameters, Initial States, etc.

The main challenges in solving PDE-constrained inverse problem involve:

- 1) High computational requirement
- 2) Ill-posedness

Proposed Approach (Training time):

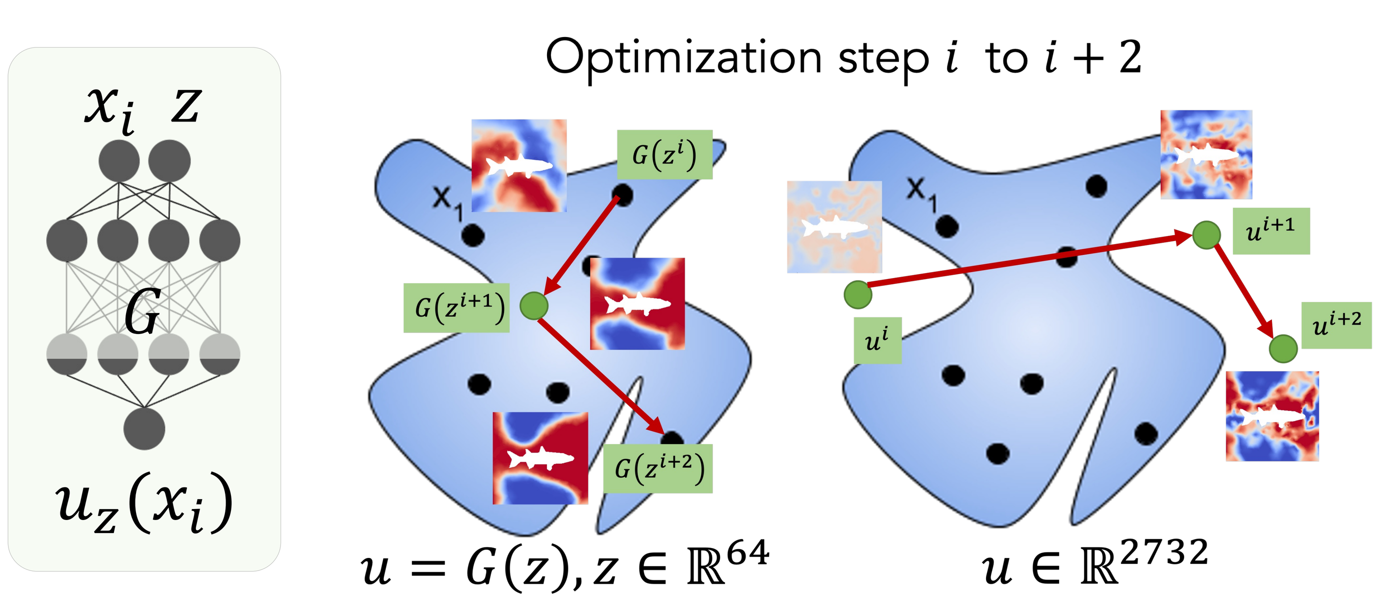
- 1) pre-training a GNN as a fast forward model



GNN provides better accuracy than the U-Net and is 35× faster than the FEM solver.

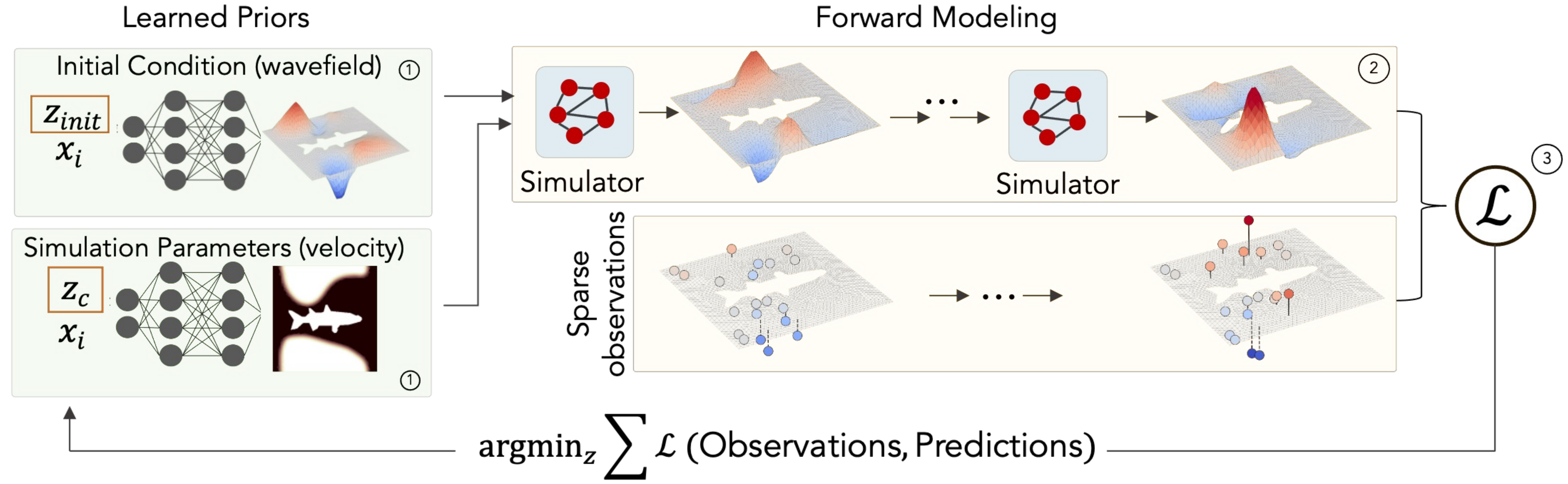
Forward Model	MSE	Runtime (s)
FEM (Irr. C.)	**3.92e-4**	3.87e1
U-Net (Reg. C.)	5.78e-3	**1.11e-1**
U-Net (Reg. F.)	2.55e-3	1.65e-1
GNN (Irr. C.)	9.73e-4	1.10e0

- 2) pre-trained generative model, G, as Prior

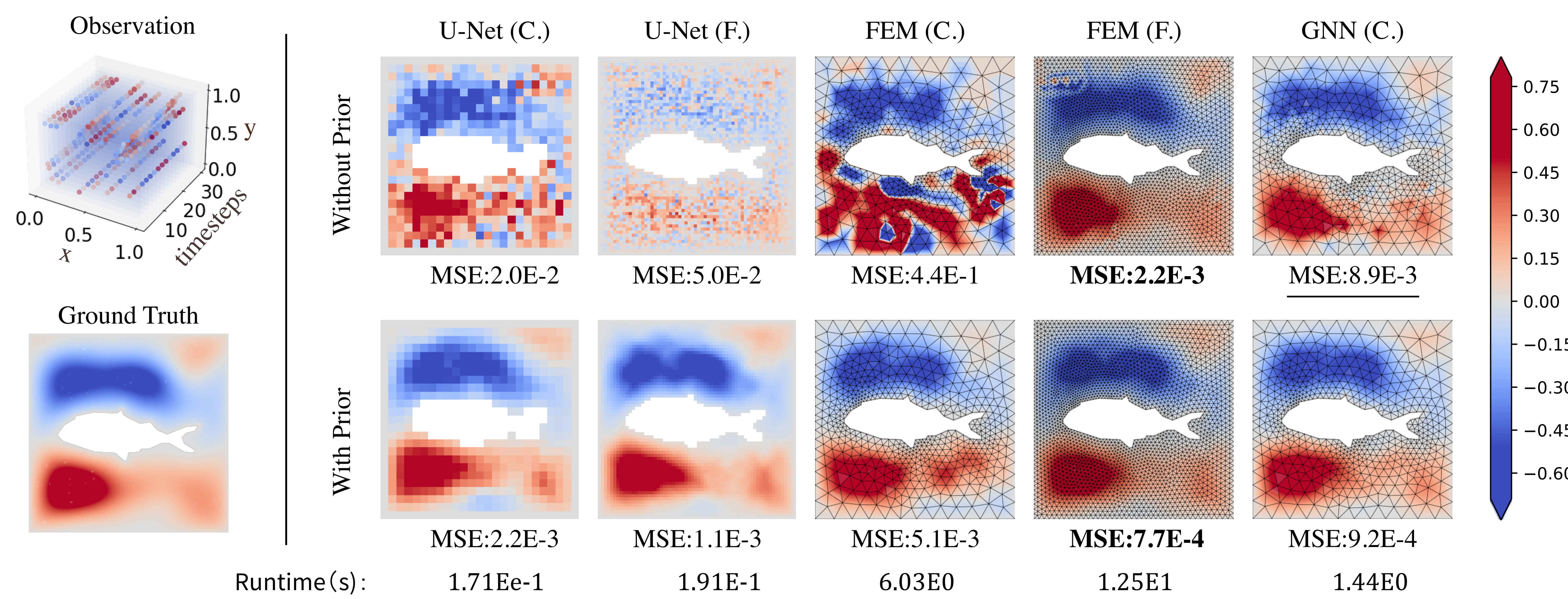


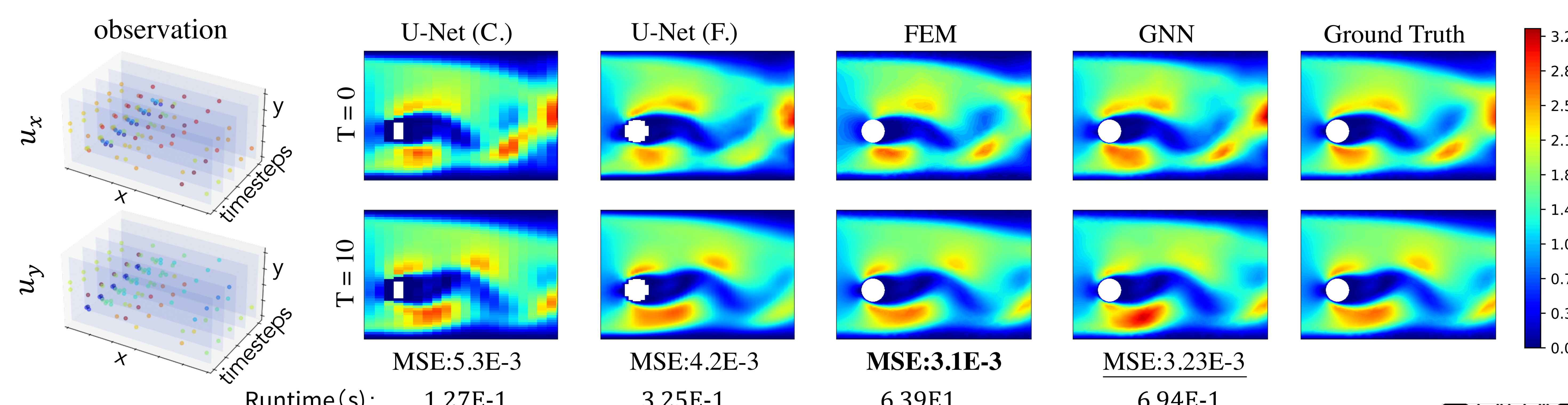
By optimizing the latent code $z \in \mathbb{R}^{64}$, we constrained the solution space to the manifold learned by G, and avoid bad local optima lie far outside the dataset distribution

Proposed Approach (Test time Pipeline):



Results





Checkout our project page for more: https://cyanzhao42.github.io/LearnInverseProblem